University of Toronto
Operations Research Group

Waste to Riches

Description

Part 1

The year is 2100 and space debris orbiting in the upper atmosphere has become a grave concern. The debris is uniformly distributed between 500km and 2000km above the Earth's surface but the density and volume of the debris particles vary at different heights. The density ρ of debris takes on the distribution:

$$\rho = 2e^{-((h-850/250))^2} \left[\frac{kg}{km^3}\right]$$

While the volume v of the debris can be modeled by

$$v = \frac{1}{2}\cos^2\frac{2\pi(h-250)}{250} \quad [km^3]$$

Where h is a variable representing the number of km above Earth's surface. To remove this debris, you have developed a "Garbage Rocket" to collect the debris and take it far away from Earth. Unfortunately, your rocket can only collect debris from one height following a launch. On the first launch the objective is to take away as much mass as possible. At what height h should the Garbage Rocket begin collecting debris? **This figure should be reported in km to 6 significant figures.**

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Part 2

After several successful debris removal missions, scientists have uncovered bands of minerals in the atmosphere. After developing a reputation as a master space junk remover, you are charged with finding the best way to collect these minerals. To help fund the mission you must maximize the value of your load, while also collecting a minimum quantity of two unknown elements for research purposes.

Mineral	Salvage Price Per Unit of Mineral	Unit of Fuel Expended Per Unit of Mineral	Minimum Units
Silver	$25	$\sin^2 \frac{\pi h}{500}$	0
Gold	$1500	$\sin^2 \frac{\pi h}{1000}$	0
Diamond	$2000	$\sin^2 \frac{\pi h}{1500}$	0
Unknown A	$0		2100
Unknown B	$0	$\sin^2 \frac{\pi h}{2500}$	1900

Your rocket has 2000 units of fuel and requires one unit of fuel for every kilometer is rises (fractional kilometers are allowed). The rocket can begin collecting minerals at any height h but once it begins collecting it may not rise again and may only descent once it runs out of fuel. During the rise and descent, the rocket cannot collect any minerals. The rocket also sustains $2750 worth of damage for every unit of fuel it expends while collecting minerals, what is the optimal height for collecting minerals? **This height should be reported in km to 6 significant figures.**

Solution Format

Submit a txt file with the height where the mass of space debris is largest. This should be followed by a space with a second value representing the best altitude for collecting minerals.

Sample Solution

The mass of space debris is largest at a height above the earth surface 503.234 km, and the best altitude for collecting minerals is 1230.23 km above the earth then you should submit

504.234 1230.23